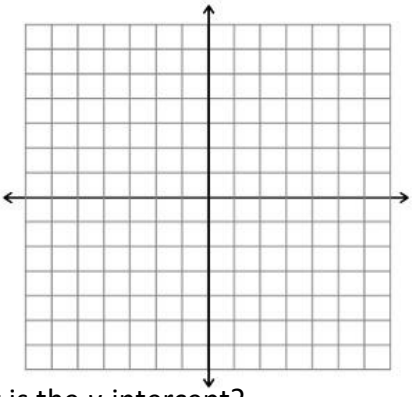
Exponential Functions

General Form: $\boldsymbol{y = ab^x}$, where $a \neq 0, b \neq 1$ and $b > 0$.

- *b* is the base and is a constant
- Domain is set of all reals; range $y > 0$; *y*-intercept $(0, a)$
- *x* is the independent variable with domain of *x* is set of all reals
- *Exponential Growth:* $a > 0$ and $b > 1$
- *Exponential Decay:* $a > 0$ and $0 < b < 1$
- Line $y = 0$ is called an *asymptote*.
- *b* – If Growth it's the *growth factor*; If Decay it's the *decay factor*

Examples:

1. What is the graph of $y = 3^x$?



2. Identify $y = 0.7^x$ as an example of exponential growth or decay. What is the *y*-intercept?

Another Model for Exponential Growth and Decay: $\boldsymbol{A(t) = a(1 + \frac{r}{n})^{nt}}$

$A(t)$ is amount after *t* time periods
a is the initial amount
r is the rate of growth or decay
t is number of time periods

- Often used when doing financial/banking questions

Examples:

1. You buy a savings bond for $25 that pays a yearly interest rate of 4.2%. What will the savings bond be worth after fifteen years?

2. You open a savings account that pays 4.5% annual interest. If your initial investment is $300 and you make no additional deposits or withdrawals, how many years will it take for the account to grow to at least $500? (Calculator Required – Table)

Natural base exponential functions: Exponential functions base *e*. *e* is a number approximately 2.71828…

- Base *e* exponential functions are found in many applied real-world applications (biology, chemistry, banking, etc.)
- *Continuously Compounded Interest*
    - $A(t) = Pe^{rt}$

Graphing characteristics same as other shifts, etc.

Examples

1. How does the graph of $y = -\frac{1}{2} \cdot 4^x$ compare to the graph of the parent function?

2. How does the graph of $y = 3^{(x+1)}$ compare to the graph of the parent function?

3. Some insects reproduce exponentially. The chart shows the population of roaches in a colony at 36-day intervals. On what day will the colony reach 50,000,000 roaches? (Hint – ExpReg)

Day	**Number of Roaches**
1	50
37	1125
73	25,290
109	569,025
145	12,803,040

4. What is the value of $2e^6$? (Calculator)

5. You have $1500 in a bank account that pays 4.5% annual interest compounded continuously. How much will you have in the account after 15 years? Round to the nearest dollar.