Functions

Operations
- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \ g(x) \neq 0$
- $(f \circ g)(x) = f(g(x)) -$ COMPOSITE FUNCTION

Domains: Consist of the $x -$ values that are in the domains for *both f* and *g*. Also, the domain of the quotient function does not contain any $x -$ value for which $g(x) = 0$.

Examples:
- Let $f(x) = 5x^3 + 1$ and $g(x) = x^2 - 4$. What are $f + g$ and $f - g$? What are the domains?

- Let $f(x) = x^2 + x - 6$ and $g(x) = x - 2$. What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

- Let $f(x) = x^2 + 1$ and $g(x) = x - 2$. What is $(g \circ f)(-2)$?

- For same functions in previous example, find $(f \circ g)(x), g(f(x)), (f \circ f)(3)$

Inverses

- If a relation pairs element *a* of its domain to element *b* of its range, the **inverse relation** pairs *b* with *a*. So, if (*a, b*) is an ordered pair of a relation, then (*b, a*) is an ordered pair of its inverse.
- The graphs of a relation and its inverse are *reflections* of each other in the line *y* = *x*.
- How to determine if an expression has an inverse? How to verify two functions are inverses?
  - A 1-1 function has an inverse that is also a function.
  - Horizontal Line Test (for every *y* there is exactly one *x*)
  - $f(g(x)) = x$ and $g(f(x)) = x$ if the two functions are inverses of each other.
    - If f and f^{-1} are inverse functions, then $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for *x* in the domain of f and f^{-1}, respectively.

- To determine inverse of a function:
  - Replace $f(x)$ with *y*, if needed.
  - Switch *x* and *y*.
  - Solve for "new" *y*. This is your inverse function $f^{-1}(x)$. "*f* inverse" "inverse of *f*"

Examples

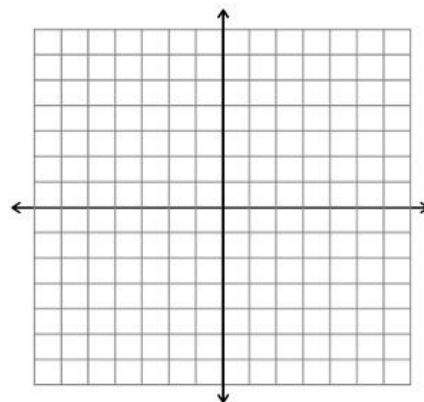- Relation *v*

x	-2	-1	0	1
y	2	0	3	0

  - What is the inverse of relation *v*?
  - What are the graphs of *v* and its inverse?
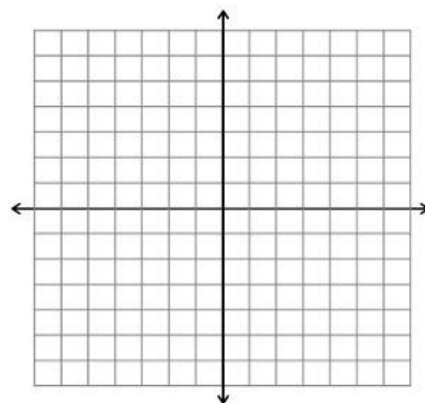
- What is the inverse of the relation described by $y = 5x^2 + 2$? Graph both f and f^{-1}.

- Consider the function $g(x) = -\frac{2}{3}x + 7$.
  - What are the domain and range of g?
  - What is the g^{-1}, the inverse of g?
  - What are the domain and range of g^{-1}?
  - Is g^{-1} a function? Explain?

- For $h(x) = \frac{3}{2-x}$, what is each of the following?
  - $h^{-1}(x)$
  - $(h \circ h^{-1})(2)$
  - $(h^{-1} \circ h)(2)$