Determinants and Inverses

Square Matrix: Matrix with same number of rows and columns
- All have multiplicative identity matrix
- All do NOT have multiplicative inverses
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- If A and B are square matrices and $AB = BA = I$, then B is the **multiplicative inverse matrix** of A, and is written A^{-1}.
- *Determinate*: All square matrices have one, and it is $det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc = detA$

  $det \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = a_1 b_2 c_3 + b_1 c_2 a_3 + C_1 a_2 b_3 - (a_3 b_2 c_1 + a_2 b_1 c_3 + a_1 b_3 c_2)$
- Can be used to find area of a triangle with vertices at $(x_1, y_1), (x_2, y_2), (x_3, y_3)$
  - Area $= \frac{1}{2}|detA|$, where $A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$
- *Inverse of 2 x 2 Matrix:*
  - If $detA = 0$; no inverse (*singular matrix*)
  - If $detA \neq 0$, then the inverse (A^{-1}) is $A^{-1} = \frac{1}{detA}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Examples:

- If $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \\ 2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{4}{3} & \frac{1}{3} & -1 \\ -1 & 0 & 1 \\ -\frac{5}{2} & -\frac{2}{3} & 2 \end{bmatrix}$, are A and B inverses?

- What is the determinant of $\begin{bmatrix} 3 & 5 & -1 \\ 1 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$?

- As part of a remodeling project, you want to paint a triangular area on a cement floor that is marked along the wall with decorative stones every meter. Using the stones as a reference, you determine the coordinates of the vertices of the area you want to paint are (4, 6), (12, 9), and (7, 11). What is the area of the triangle?

- Dose the matrix $A = \begin{bmatrix} 4 & -4 \\ -3 & 6 \end{bmatrix}$ have an inverse? If it does, what is A^{-1}?

- You stored your credit card numbers in a file after they were coded by the matrix $\begin{bmatrix} -5 & 3 \\ 3 & -7 \end{bmatrix}$. One of the numbers in the file is -1, -14, -16, -22, 10, 10, -31, -29, -15, -2, -6, 8, -32, -32, 3, 7. What is the original credit card number?