Dividing Polynomials

Long Division: Another way of factor a polynomial. Similar to numerical long division.

- Both the polynomial, $P(x)$ and divisor, $D(x)$ need to be in standard form and input zero coefficients when needed. Process stops when the degree of the remainder, $R(x)$ is less than the degree of the divisor.
- If no remainder is found, then the divisor is a factor of the polynomial.
- Sometimes beneficial to determine whether $x - a$ is a factor of the polynomial first. How do we do this?
    - If $x - a$ is a factor, then it is a *zero* of the polynomial (meaning….when you input *a* into the polynomial, it should output zero).

Synthetic Division: Yet another way of factoring a polynomial.

- Write down just the coefficients of the polynomial, in standard form (don't include variables or exponents).
- Don't forget "place-holders" if needed
- Divisor—reverse the sign of $x - a$ term.

Remainder Theorem: If you divide $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is equal to $P(a)$.

Examples:

- Use long division to divide $5x^2 + 2x + 3$ by $x + 1$. What are the quotient and remainder?

- Is $x^2 - 2$ a factor of $P(x) = x^4 - x^2 - 2$? If it is, write $P(x)$ as a product of two factors.

- Use synthetic division to divide $4x^3 - 3x^2 + 2x - 3$ by $x - 1$. What are the quotient and the remainder?

- The polynomial $x^3 + 9x^2 + 23x + 15$ expresses the volume, in cubic inches, of a box, and the length is $(x + 5)$ inches. What are the other two dimensions of the box?

- Given that $P(x) = x^4 + 6x^3 + 9x^2 + 3x - 3$, what is $P(4)$?