Rational Functions

Format: $y = \dfrac{P(x)}{Q(x)}$

- Domain: Set of all reals *except* those which make the denominator zero!!!!
- If there is no value(s) for which the denominator is zero, then the function is said to be continuous
  $y = \dfrac{x}{x^2 + 8}$
- If there is a value(s) where the denominator is zero, there is *discontinuity*.
  - *Removable Discontinuity*: Could you make the function continuous by redefining discontinuity point(s).
  $$y = \frac{(x+3)(x+2)}{(x+2)}$$
  What if you made $f(-2) = 1$? IE, do the *y* values get infinitely close to a value on both sides of the discontinuity point?
  --Occur when you factor both the numerator and the denominator, and they share a factor (the discontinuity caused by this factor is removable)
  - *Non-Removable Discontinuity*: If there is no value that you could replace in for the discontinuity point to make continuous.
  $$y = \frac{x+4}{x-2}$$

- *Asymptotes:*
  - *Vertical*: Occur when denominator of rational function is zero. Vertical asymptotes occur at the *line x =* ____

  - *Horizontal*:
    - If power of numerator greater than power of denominator: No horizontal asymptote.
    - If power of denominator greater than power of numerator: Horizontal asymptote at *line y =* ____
    - If the powers are the same, horizontal asymptote at *line* $y = \dfrac{a}{b}$ where *a* and *b* are the leading coefficients of the numerator and denominator.

  - *Slant (Oblique)*: Occur when the numerator is *exactly* one more degree than the denominator
    - Must perform long division to determine *line* where slant asymptote occurs

- Graphing
  - Plot at least one point between and one point beyond each *x* – intercept and vertical asymptote

Examples:

1. What are the domain and points of discontinuity of $y = \dfrac{x^2 + 4x + 4}{x+2}$? Are the points of discontinuity removable or non-removable? What are the *x* and *y* intercepts?

2. What are the vertical asymptotes of the graph $y = \dfrac{x-3}{(x^2-3x+2)(x^2-7x+1\ )}$?

3. What is the horizontal asymptote of the graph of $y = \dfrac{x^2+1}{-3x^2+6}$? Graph the function.