Rational Functions Notes

Rational Functions

Format: $y = \frac{P(x)}{Q(x)}$

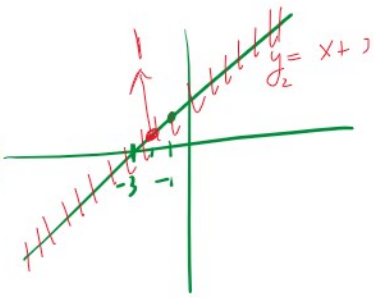
- Domain: Set of all reals *except* those which make the denominator zero!!!!
- If there is no value(s) for which the denominator is zero, then the function is said to be continuous
  $y = \frac{x}{x^2+8}$
- If there is a value(s) where the denominator is zero, there is *discontinuity*.
  - *Removable Discontinuity*: Could you make the function continuous by redefining discontinuity point(s).

    $$y = \frac{(x+3)(x+2)}{(x+2)}$$

    Discontinuity: $x = -2$

    What if you made $f(-2) = 1$? IE, do the *y* values get infinitely close to a value on both sides of the discontinuity point?

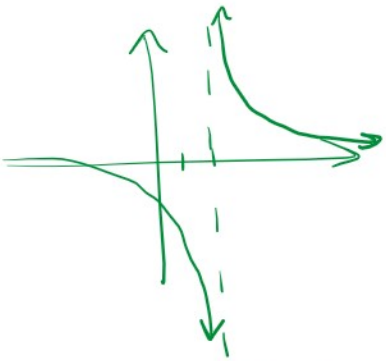
    --Occur when you factor both the numerator and the denominator, and they share a factor (the discontinuity caused by this factor is removable)



  - *Non-Removable Discontinuity*: If there is no value that you could replace in for the discontinuity point to make continuous. Vertical Asymptotes

    $$y = \frac{x+4}{x-2}$$

-2 ERROR



- *Asymptotes:*
  - *Vertical*: Occur when denominator of rational function is zero. Vertical asymptotes occur at the *line* $x =$ ___
  - *Horizontal*:
    - If power of numerator greater than power of denominator: No horizontal asymptote.
    - If power of denominator greater than power of numerator: Horizontal asymptote at *line* $y =$ 0
    - If the powers are the same, horizontal asymptote at *line* $y = \frac{a}{b}$ where *a* and *b* are the leading coefficients of the numerator and denominator.

$y = \frac{x^2-3}{x+1}$ $y = \frac{x+1}{x^2-3}$

$y = \frac{3x^3-2x^2+7x-3}{4x^3+2x^2-3x+5}$ $y = \frac{3}{4}$

  - *Slant (Oblique)*: Occur when the numerator is *exactly* one more degree than the denominator
    - Must perform long division to determine *line* where slant asymptote occurs

- Graphing
  - Plot at least one point between and one point beyond each *x* – intercept and vertical asymptote

As $x \to a$, $f(x) \to \infty$ or $f(x) \to -\infty$

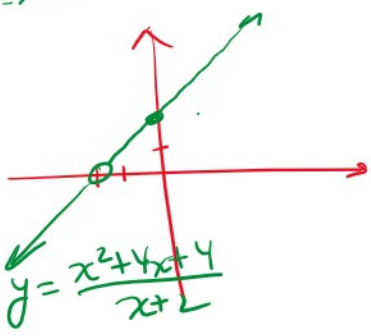
Examples: y-intercepts: $(0, \_\_\_)$ $(\_, 0)$

1. What are the domain and points of discontinuity of $y = \frac{x^2+4x+4}{x+2}$? Are the points of discontinuity removable or non-removable? What are the *x* and *y* intercepts?

Domain: $\{\mathbb{R} \mid x \neq -2\}$
Discontinuities: $x = -2$
Removable Discontinuity

$y = \frac{(x+2)(x+2)}{(x+2)}$

$y = x+2$

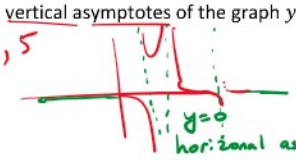


x-intercepts: When does $y = 0$? For a fraction this is when Numerator equals 0
$(x+2)(x+2) \stackrel{?}{=} 0$ $x = -2$
Since $x = -2$ is NOT in domain, There isn't a x-intercept.
y-intercepts: let $x = 0$ $(0, 2)$ $y = \frac{0^2+4(0)+4}{0+2} = \frac{4}{2}$

$y = \frac{x^2+4x+4}{x+2}$

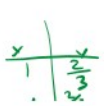
2. What are the vertical asymptotes of the graph $y = \frac{(x-3)}{(x^2-3x+2)(x^2-7x+10)}$?

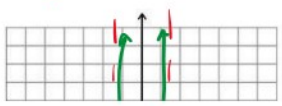
$x = 2, 1, 5$
$(x-2)(x-1)(x-5)(x-2)$
Discontinuities: $x = 2, 1, 5$



3. What is the horizontal asymptote of the graph of $y = \frac{x^2+1}{-3x^2+6}$? Graph the function.

$y = -\frac{1}{3}$ $(0, \frac{1}{6})$





d = 3

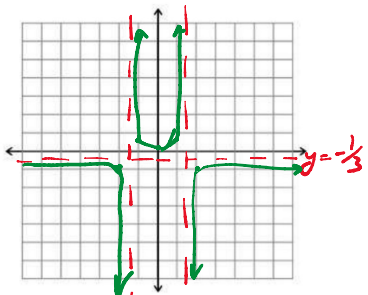
x	y
1	$\frac{2}{3}$
-1	$\frac{2}{3}$
2	$\frac{5}{6}$
-2	$\frac{5}{6}$

Vertical Asymptote?

$-3x^2+6=0$

$x^2=\frac{-6}{-3}=2$

$x=\pm\sqrt{2}\approx\pm 1.4$



$x=-\sqrt{2}$ $x=\sqrt{2}$