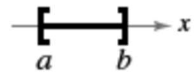
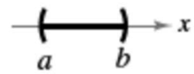
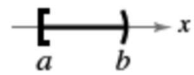
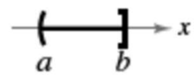
Turbo Algebra Review

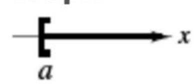
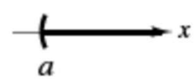
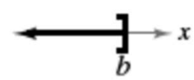
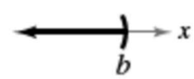
- Interval Notation
  - Bounded – Both endpoints are defined

Bounded Intervals on the Real Number Line

Notation	Interval Type	Inequality	Graph
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \leq x < b$	
$(a, b]$		$a < x \leq b$	

  - Unbounded – One or both endpoints are infinity (unbounded)

Unbounded Intervals on the Real Number Line

Notation	Interval Type	Inequality	Graph
$[a, \infty)$		$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \leq b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

  - Examples:
    - $-3 \leq x \leq 5$
    - $x \geq 2$
    - $x \leq -1$

- Fractions:
  - Common Denominator when adding/subtracting
  - Always straight across when simplifying/combining
  - $\frac{N}{O}$
  - Examples:
    - $\frac{2}{3} - \frac{1}{9} =$
    - $\frac{3}{4} \cdot \frac{7}{15} =$

- Exponents

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

Property	Example
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$
2. $\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^7}{x^4} = x^{7-4} = x^3$
3. $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$y^{-4} = \frac{1}{y^4} = \left(\frac{1}{y}\right)^4$
4. $a^0 = 1$	$(x^2+1)^0 = 1$
5. $(ab)^m = a^m b^m$	$(5x)^3 = 5^3 x^3 = 125x^3$
6. $(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \frac{1}{y^{12}}$
7. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{x}\right)^3 = \frac{2^3}{x^3} = \frac{8}{x^3}$
8. $\lvert a^2 \rvert = \lvert a \rvert^2 = a^2$	$\lvert (-2)^2 \rvert = \lvert -2 \rvert^2 = 2^2 = 4 = (-2)^2$

  - PEMDAS – She limps from left to right
  - Anything to the zero power equals what???
  - Examples:
    - $(-2x^{-1})^3$
    - $\frac{-(2x^3)^2}{y}$
    - $(2x^{-2}y^3)(-x^4y)$

- Radicals

Properties of Radicals

Let a and b be real numbers, variables, or algebraic expressions such that the roots below are real numbers, and let m and n be positive integers.

Property	Example
1. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$\sqrt[3]{8^2} = (\sqrt[3]{8})^2 = (2)^2 = 4$
2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{5} \cdot \sqrt{7} = \sqrt{5 \cdot 7} = \sqrt{35}$
3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}, b \neq 0$	$\dfrac{\sqrt[4]{27}}{\sqrt[4]{9}} = \sqrt[4]{\dfrac{27}{9}} = \sqrt[4]{3}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{10}} = \sqrt[6]{10}$
5. $(\sqrt[n]{a})^n = a$	$(\sqrt{3})^2 = 3$
6. For n even, $\sqrt[n]{a^n} = \|a\|$.	$\sqrt{(-12)^2} = \|-12\| = 12$
For n odd, $\sqrt[n]{a^n} = a$.	$\sqrt[3]{(-12)^3} = -12$

- Examples
  - Simplify: $\sqrt{3} \cdot \sqrt{8} =$
  - Simplify: $\sqrt[3]{x^2} \cdot \sqrt[3]{x} =$
  - Simplify: $3\sqrt{8} + \sqrt{18} =$
  - Rationalize: $\frac{4}{3\sqrt{2}} =$

- Continuation of radical properties:
  - $a^{\frac{1}{n}} = \sqrt[n]{a}$
  - $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$
  - Examples:
    - Write in Exponent Form: $4x\sqrt[3]{x^2}$

- Write in Radical Form: $-2x^{\frac{1}{3}}y^{\frac{2}{3}}$

- Polynomials:
  - Adding and Subtracting: PEMDAS, and like terms get combined
    - Example: $(4\mathrm{y}^2 - 3) - (-7\mathrm{y}^2 + 9)$

- Multiplying:
  - FOIL (FIRST OUTER INNNER LAST)
  - Example: (7*x* - 3)(3*x* + 2)

  - Example: $(2\mathrm{x} - 3)^2$

- Factoring Polynomials: Used to find zero's, simplifying polynomials, and solving polynomials
  - SEE WHAT YOU CAN PULL OUT! (Common factor--GCF!)
  - Perfect square trinomials have easy format to factor:
    - $a^2 + 2ab + b^2 = (a + b)^2$
    - $a^2 - 2ab + b^2 = (a - b)^2$
    - $a^2 - b^2 = (a - b)(a + b)$
  - Sum or Difference of perfect cubes
    - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
    - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  - *Perfect Square Trinomials*: $a^2 + 2ab + b^2 = (a + b)^2$ AND $a^2 - 2ab + b^2 = (a - b)^2$
  - *Difference of Perfect Squares*: $a^2 - b^2 = (a - b)(a + b)$
    - NOTICE: No middle term! EX: $4x^2 - 25$
  - Hint: If *a* = -1, you can pull out -1 factor to enable easy factoring.
  - Don't forget what solving a quadratic means. Represents graphically.
  - Group like factors, if required (ie, $(x - 1)(x - 1) = (x - 1)^2$
  - EX: Factor: $x^2 + 7x + 6$

- EX: What is the expression $4x^2 - 28x$ in factored form?

- EX: What is the expression in factored form? $2x^2 + 5x - 12$

- EX: What is $x^2 - 12x + 36$ in factored form?

- EX: What is $81x^2 - 100$ in factored form?

- What are the *solutions* of the quadratic equation: $x^2 + 3x - 18 = 0$

- Solve: $10x^2 + 2x - 46 = x - 4$

- EX: What are the zero's of: $5x^2 - 8 = 18x$

- EX: Factor: $x^3 - 64$

- *Rational Expression*: Quotient of two polynomials.
    - *Simplest Form*: When both numerator and denominator are polynomials that no longer share any common divisors/factors.
    - *Domain*: Comes from non-simplified form
    - *Adding/Subtracting*: Find LCM of denominator.
        - Factor expressions completely.
        - LCM of the denominator (LCD) is product of all prime factors (unique) to their highest power

    - Examples:
        - What is $\frac{9x^2+6}{36x+24}$ in simplest form? State any restrictions on the variable.
        - What is the product $\frac{x^2-3x+2}{x+2} \cdot \frac{x^2-36}{x^2+5x-6}$ in simplest form. State any restrictions on the variable.
        - What is the quotient $\frac{6x-3x^2}{36-x^2} \div \frac{x^3-x^2-2x}{x^2-5x-6}$ in simplest form? State any restrictions on the variable.

- What is the LCM of $x^2 + 4x - 12$ and $x^2 - 6x + 8$?

- What is the sum $\frac{4}{x^2+3x} + \frac{x-2}{x^2+6x+9}$ in simplest form? State any restrictions on the variable.

  - What is the difference $\frac{x+1}{x^2+2x-8} - \frac{x}{4x-8}$ in simplest form? State any restrictions on the variable.